



Fermi National Accelerator Laboratory

FERMILAB-Conf-93/117-T

A Note on 'Hot Spot' Hunting in Deep-Inelastic Scattering

E. LAENEN¹ AND E. LEVIN²

*Fermi National Accelerator Laboratory,
P.O. Box 500, MS 106
Batavia, Illinois 60510*

*Talk presented³ at UK Phenomenology Workshop
'HERA - the new frontier for QCD', Durham
March, 1993*

Abstract

We consider the inclusive cross section for jet production with large transverse momentum in deep-inelastic scattering. This process has been proposed as a probe of small- x physics, particularly the measurement of 'hot spots' inside the proton. We present a numerical calculation of this process, taking into account a larger phase space. The theoretical reliability as well as phenomenological uncertainties of the calculation are discussed.

¹email: eric@fnth010.fnal.gov

²On leave from St. Petersburg Nuclear Physics Institute, 188350 Gatchina, St. Petersburg, Russia

e-mail: levin@fnal.fnal.gov, FNAL::LEVIN

³by E. Levin



1 Introduction.

In order to study the main properties of low x_B deep-inelastic scattering processes, Mueller suggested in [1] an experiment in which all anticipated new phenomena in this kinematical region should have a large effect. His idea was to measure the inclusive production of a gluon jet with a transverse momentum $k_{j,t}$ very close to the photon virtuality Q and with a fraction of energy x_j as close to one as is feasible, so that the ratio x_B/x_j can be small. In this case,

- the cross section of the process can be calculated within the framework of perturbative QCD, if $k_t^2 \simeq Q^2 \gg \Lambda_{\text{QCD}}^2$.
- the dependence of the cross section on x_B is governed by low x_B gluon emission which can be described by the BFKL evolution equation [2]. This is in contrast with the usual GLAP [3] approach, in which the cross section is described by the simple Born diagram of Fig.1a. and turns out to be constant with respect to x_B .
- the scale of the shadowing corrections is determined by the size of the ‘hot spot’, namely $R \simeq 1/k_{j,t}$ and they are expected to be large (see ref. [4]).

Numerical estimates for this process have been performed in a series of papers [5]-[9], but, in our view, the matter has not been settled yet (cf. the strong dependence on an infrared cut-off in the BFKL equation with running coupling, for values of k_t^2 and Q^2 that are smaller than about 50 GeV² [7]-[9]).

In this paper we reconsider the theoretical formulae for the cross section and present our numerical estimates in studying the small- x and infrared behavior of the one-jet inclusive cross section. We will not consider any shadowing corrections.

2 The basic formula.

The correct formula for inclusive one-jet production in the region of small x_B was given in ref. [11]. In terms of a differential structure function involving the jet variables x_j and k_t^2 (resp. the longitudinal momentum fraction of the

jet and its tranverse momentum squared), it looks as follows for gluon jet production

$$\begin{aligned}
\frac{dF_2(x_B, Q^2; x_j, k_t^2)}{d \ln x_j dk_t^2} &= \frac{3N_c}{\pi k_t^2} \int \frac{d^2 \mathbf{k}_{1,t} d^2 \mathbf{k}_{2,t}}{\pi} \cdot \alpha_S(\min\{k_{1,t}^2, k_{2,t}^2, k_t^2\}) \\
&\quad \phi_B\left(\frac{x_B}{x_j}, k_{1,t}^2, Q^2\right) \phi_G(x_j, k_{2,t}^2) \cdot \delta^{(2)}(\mathbf{k}_t - \mathbf{k}_{1,t} - \mathbf{k}_{2,t}) \\
&= \frac{3N_c}{\pi k_t^2} \int \frac{dk_{1,t}^2 dk_{2,t}^2}{\sqrt{-\lambda(k_{1,t}^2, k_{2,t}^2, k_t^2)}} \cdot \alpha_S(\min\{k_{1,t}^2, k_{2,t}^2, k_t^2\}) \\
&\quad \phi_B\left(\frac{x_B}{x_j}, k_{1,t}^2, Q^2\right) \phi_G(x_j, k_{2,t}^2), \tag{2.1}
\end{aligned}$$

where all notation is explained in Fig.1b., $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$, and ϕ_G is the gluon density, related to the gluon distribution function $xG(x, Q^2)$ by

$$\frac{d\alpha_S(k^2)xG(x, k^2)}{dk^2} = \phi_G(x, k^2)\alpha_S(k^2). \tag{2.2}$$

As was shown in ref. [10], eq. (2.1) can be reduced in the double logarithm approximation (DLA) of perturbative QCD to the expression of eq.(51) in [11], where the inclusive production in deep-inelastic scattering was studied in detail within this approximation. (see also [10]). If we integrate in eq. (2.1) only the part of phase space where $k_{2,t}^2 \ll k_{1,t}^2$ ($k_{1,t}^2 \simeq k_t^2$) ('small' phase space) we can rewrite eq. (2.1) in the form:

$$\begin{aligned}
\frac{dF_2}{d \ln x_j dk_t^2} &= \frac{3N_c}{\pi k_t^2} \int dk_{2,t}^2 \cdot \alpha_S(k_{2,t}^2) \phi_B\left(\frac{x_B}{x_j}, k_t^2, Q^2\right) \phi_G(x_j, k_{2,t}^2) \\
&= \frac{3N_c}{\pi k_t^2} \alpha_S(k_t^2) \phi_B\left(\frac{x_B}{x_j}, k_t^2, Q^2\right) x_j G(x_j, k_t^2). \tag{2.3}
\end{aligned}$$

It is this equation that was used in all previous numerical estimates of the inclusive jet production [5]-[9]. Here we calculate the differential structure function without any restriction on the region of integration ('large' phase space). In comparing the two we will find an enhancement of the cross section due to the larger phase space of about 80 %.

3 Calculational procedure.

The functions ϕ in eq. (2.1) are solutions of the BFKL-equation [2],

$$\frac{\partial \phi(x, k^2)}{\partial \ln(1/x)} = \frac{3\alpha_S}{\pi} \int_{k_0^2}^{\infty} dk'^2 \left\{ \frac{\phi(x, k'^2) - \phi(x, k^2)k^2/k'^2}{|k'^2 - k^2|} + \frac{k^2}{k'^2} \frac{\phi(x, k'^2)}{(4k'^4 + k^4)^{(1/2)}} \right\}. \quad (3.1)$$

We now discuss aspects of our procedure of solving this equation.

We chose different initial conditions for the functions $\phi_B(\frac{x_B}{x_j}, k_i^2, Q^2)$ and $\phi_G(x_j, k_i^2)$ in (2.1). For $\phi_B(\frac{x_B}{x_j}, k_i^2, Q^2)$ we used the same initial condition as in ref. [7], namely at $z_0 = x_B/x_j = 10^{-1}$

$$\phi_B(z_0, k_i^2, q^2) = \frac{F_0(z_0, k_i^2, Q^2)}{k^2} \simeq \frac{F_0(k^2, Q^2)}{k^2}, \quad (3.2)$$

where the function F_0 , related to the quark box diagram, was calculated in refs. [5]-[7]. For $\phi_G(x_j, k_i^2)$ we have to reconstruct the initial condition from experimental data. However, in order to solve the BFKL-equation we need to know the behavior of ϕ at any value of k_i^2 for some fixed x , even at $k_i^2 \rightarrow 0$. To accomplish this, we used the following procedure to describe the low k_i^2 behavior of $x_j G(x_j, k_i^2)$:

$$x_j G(x_j, k^2) \rightarrow \frac{k^2}{k^2 + q_0^2} \tilde{x}_j G(\tilde{x}_j, \tilde{k}^2) \quad (3.3)$$

where $\tilde{k}^2 = k^2 + q_0^2$, $\tilde{x} = x/(x + k^2/\tilde{k}^2(1 - x))$. We used the mapping (3.3) because (1) this parametrization ensures $x_j G(x_j, k^2) \simeq k^2$ as $k^2 \rightarrow 0$ and such a behavior is the direct consequence of the gauge invariance of QCD, and (2) it works for the case of F_2 (see refs. [12]-[13]).

For the function $\tilde{x}_j G(\tilde{x}_j, \tilde{k}^2)$ we used a fit ⁴ to the data set from the CTEQ collaboration [14] down to Q^2 values of 1 GeV². The initial condition for ϕ_G was then constructed according to (2.2) (we used a fixed coupling), at a value $x = 10^{-2}$, where, as in [7], instead of $xG(x, k^2)$ we used the effective density $xG(x, k^2) + \frac{4}{9} \sum_{f=1}^4 x[q_f(x, k^2) + \bar{q}_f(x, k^2)]$.

We solved⁵ the BFKL equation (3.1) with a fixed coupling $\alpha_S = \alpha_S(Q^2)$ because this equation only sums the leading logs $(\alpha_S \ln(1/x))^n$, and not the

⁴We are grateful to J. Botts for making this fit, and discussions relating to it.

⁵We are grateful to J. Kwiecinski for sending us his program to solve the BFKL equation numerically.

subleading ones. The latter remains an unsolved problem. However, we made a rough estimate of how important the corrections from a running α_S could be by calculating an average α_S

$$\langle \alpha_S \rangle_i = \frac{\int dk^2 \alpha_S(k^2) \phi_i(x, k^2)}{\int dk^2 \phi_i(x, k^2)} \quad , \quad i = B, G \quad (3.4)$$

The dependence of $\langle \alpha_S \rangle_i$ on x_B showed that we could use $\alpha_S = \alpha_S(Q^2)$ as a good first approximation. For example, for $k^2 = Q^2 = 10 \text{ GeV}^2$, we found $\alpha_S(Q^2) \simeq 0.2$, and $\langle \alpha_S \rangle_i \simeq 0.15$ for $x < 10^{-2}$ for both $i = B, G$.

A much more detailed study of this problem can be found in [9].

The last point we discuss is the infrared cut-off k_0^2 in eq. (3.1). In principle eq. (3.1) is infrared stable and one can take $k_0^2 = 0$. However, in order to further investigate the dependence of the solution of (3.1) on small momenta we preferred to introduce a cut-off k_0^2 and see how much the answer depends on its value.

4 Results and conclusions.

The results are shown in Figs.2a and 2b. for the values $k^2 = Q^2 = 10 \text{ GeV}^2$, $x_j = 10^{-2}$. The first observation we make from Fig.2a is that the answer for (2.1) when one includes the correct ('large') phase space increases the results of previous calculations (small phase space) by about 80 %. Furthermore, the dependence on q_0^2 is visible, but does not compensate for the difference between small and large phase space. A similar conclusion we found to hold for the dependence on the infrared cut-off k_0^2 .

The most discouraging result is shown in Fig.2b. Here we plot the normalized differential structure function

$$R_2 = \frac{1}{F_2(x_B, Q^2)} \frac{dF_2(x_B, Q^2; x_j, k^2)}{d \ln x_j dk^2} \quad (4.1)$$

where we took $F_2(x, Q^2) = \int dk^2 \phi_B(\frac{x_B}{x_j}, k^2, Q^2) \phi_G(x_j, k^2)$. One notes that this ratio seems to be independent of x_B . This would seem to indicate that, within the approximations made, this special environment does not seem to be much better for measuring small-x effects than a direct measurement of F_2 . This feature persisted when we took $F_2(x, Q^2)$ constructed from the MRS D' distributions [15].

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Figure Captions

Fig.1a. Born diagram for single gluon jet production in deep-inelastic scattering.

Fig.1b. Single-jet production in deep-inelastic scattering in the large phase space case.

Fig.2a. Plot of $\frac{dF_2}{d \ln x_j dk^2}$ vs. x_B . The value of x_j is 0.01 and we took $k^2 = Q^2 = 10 \text{ GeV}^2$. The solid line corresponds to the small phase space case, the three remaining to the large phase space case with three different values of q_0^2 , namely $q_0^2 = 1 \text{ GeV}^2$ (long-dashed), $q_0^2 = 2 \text{ GeV}^2$ (short-dashed) and $q_0^2 = 4 \text{ GeV}^2$ (dotted).

Fig.2b. The ratio R_2 (4.1) for three values of q_0^2 in the large phase space case. The notation is the same as in Fig.2a.

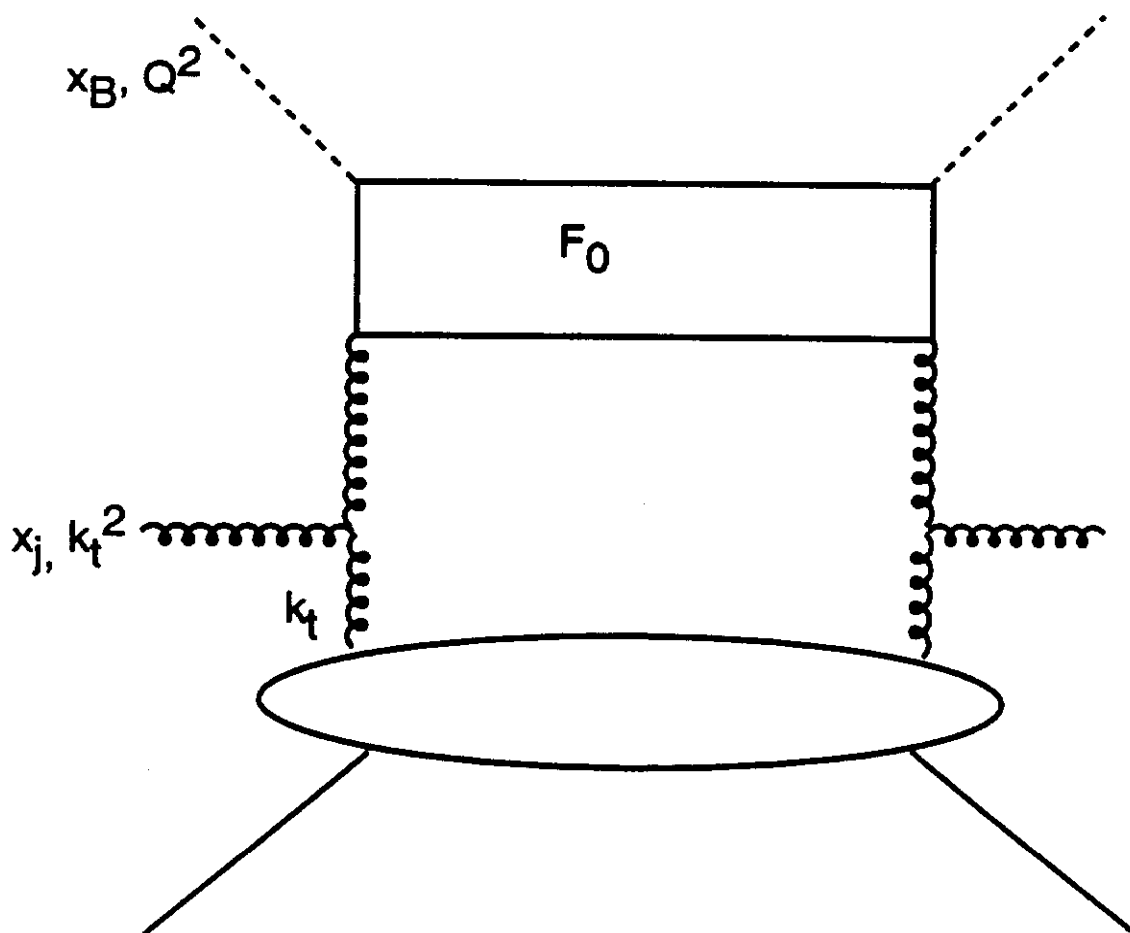


Fig. 1a

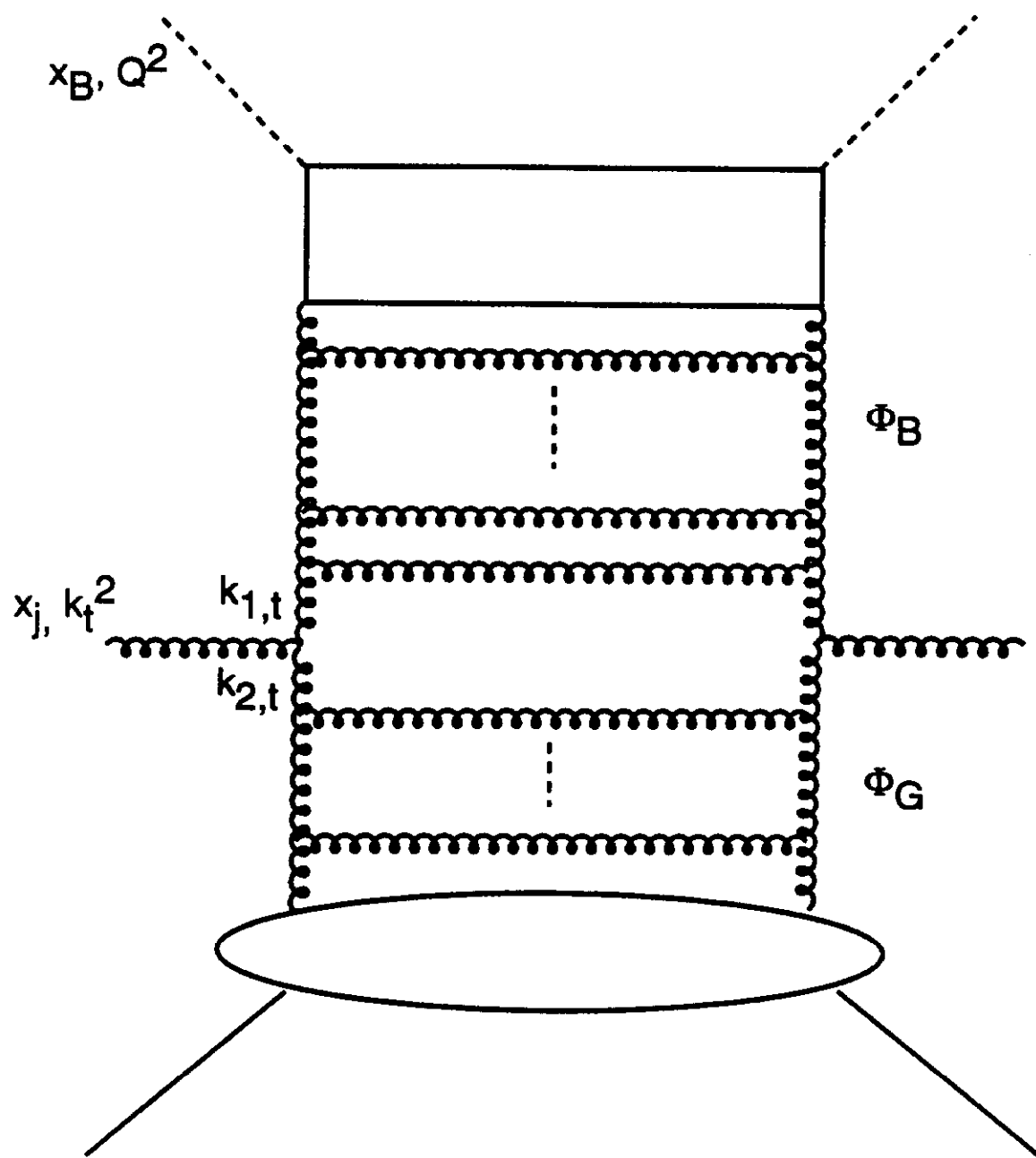


Fig. 1b

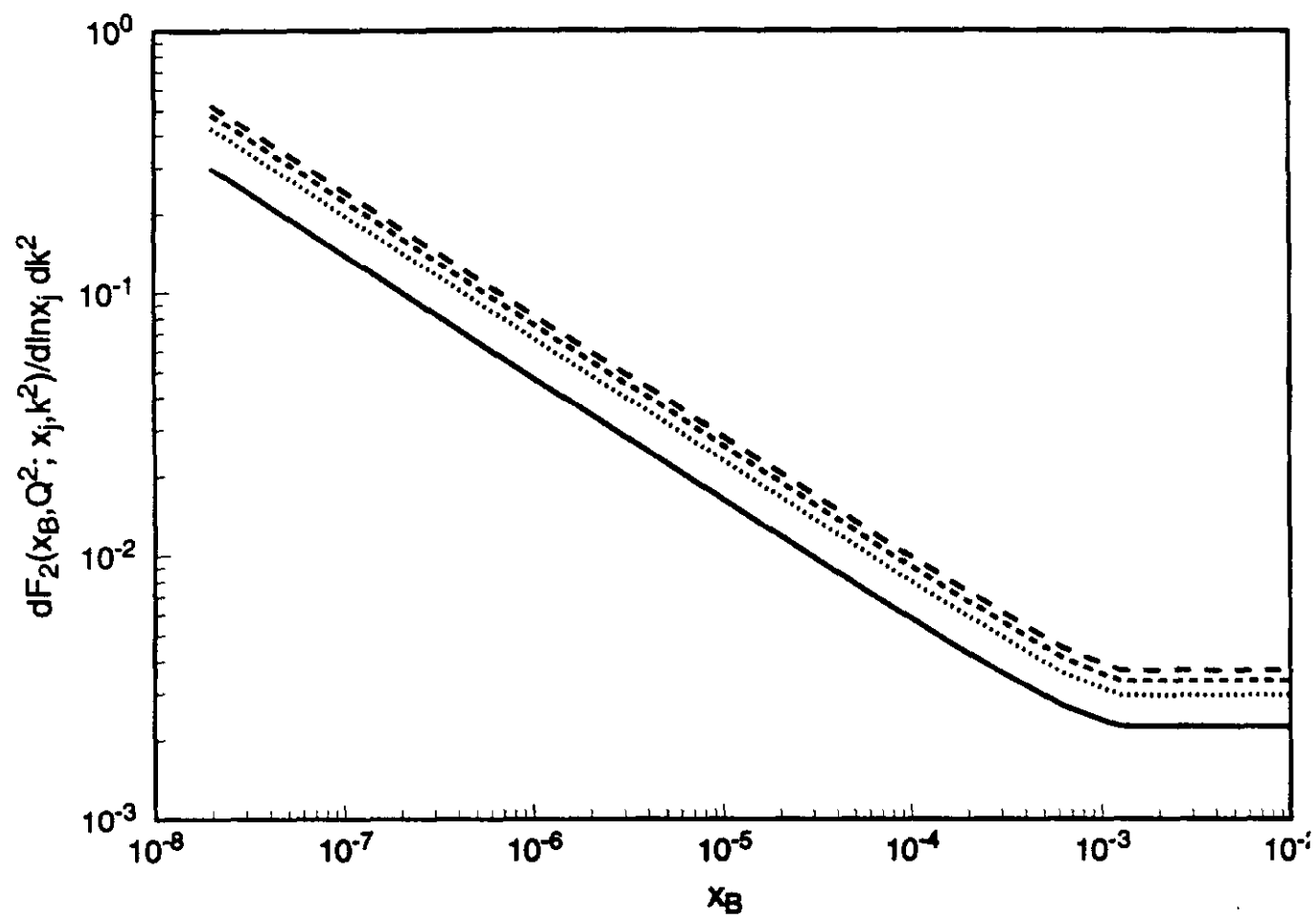


Fig. 2a

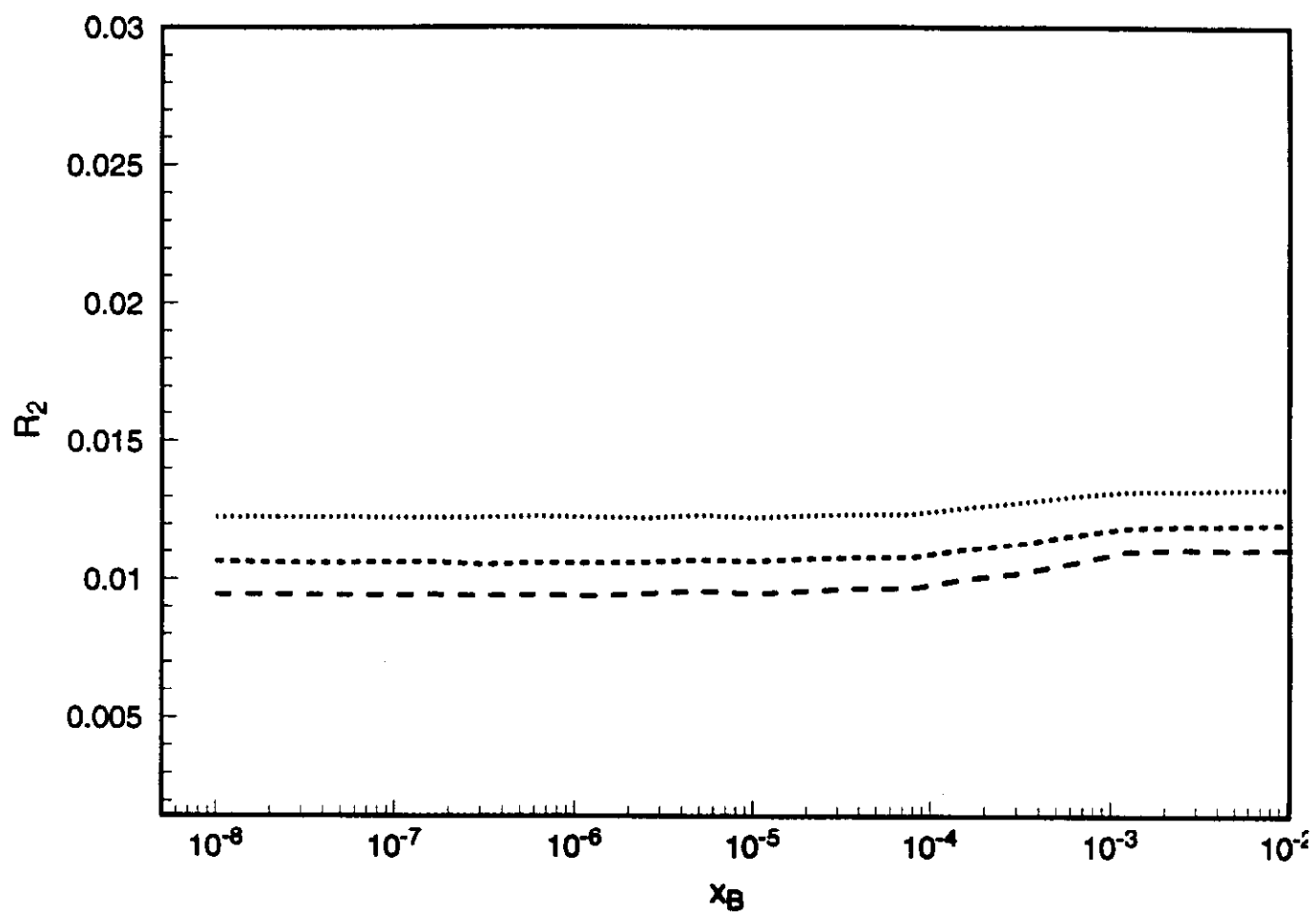


Fig. 2b